An introduction of digital technologies in the university classroom

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Resumen: Esta es una propuesta para promover una mejor comprensión de los conceptos del cálculo diferencial en un primer curso universitario, mediante la introducción de las tecnologías digitales bajo un marco didáctico y con una aproximación epistemológica. Presentaremos algunos resultados.

Abstract. We propose a way to promote a better understanding of the concepts of differential calculus in a first course for undergraduate students, by introducing digital technologies under a didactic outline in accordance with an epistemic approach.. We present some results of the experiment carried out in a University in Mexico.

Words key: differential calculus, digital technologies, didactic, epistemic

1. Introduction: Differential Calculus, the observed troubles in its study

Traditionally, the results of approval obtained in the calculus course are very low. This is evident in the engineer careers in Mexico, where the percentage of failure is higher than 70%. This same situation appears in many other institutions as well as abroad, i.e. Tall (1995), citing Anderson & Loftsgaarden, mentions that nevertheless the students are submitted to a heavy regimen of calculus exercises, the failure percentage results in between 30 and 50% (cfr. Steen, 1987; Cuevas, 1996; Baker et.al.2001).

One of the possible reasons for this failure is, undoubtedly, that traditional mathematics education and in particular, teaching of calculus, is polarized in two extremes: on the one side the subject is directed under a strong operative load deteriorating the conceptual side, and on the other, calculus education is taught under a strong formal mathematics inheritance. Both circumstances drive to a poor comprehension and application of the concepts. It is even a common belief among the students that making mathematics means making punctual operations, signs manipulation and memorizing (Skemp 1976, Orton 1983, Carpenter 1996).

For example, the teaching of differential calculus usually begins in typical statements, theorems and problems which exemplify the associated concepts, and it is mainly supported by students' algebraic knowledge and little in the geometric and visual intuition, possibly because of the difficulty in representing in paper or board a big enough number of examples giving geometric meaning to the involved calculus contents and algebra. It would be necessary to catch the development of calculus through change and variation problems from physics, which help to reinforce intuition. Nowadays, the change and variation problems are examples of application of calculus; they are studied after developing the theory and not as historically it arose. Other important reason of failures observed, in the obtained results from student's diagnostic tests, are the deficiencies in the general concept knowledge of function and of the related concepts (independent variable, dependent variable, parameter, equation). We hope to compensate these deficiencies, not with remedial courses, which never have satisfactory effects, but with the adequate use of the computing resources.

2. A historical perception

Traditionally, the ordered subjects in the first course of differential calculus at an undergraduate level are: Real numbers; Real functions; Concept of limit; Derivate of a real function in a point; Rules and results of derivation; Derivate applications . Nevertheless history shows that the development of the concepts is quite different form the way in which the curricular contents of the calculus course are currently presented. History on the XVIII and XIX centuries teaches us for example, that the complete study of the real numbers was tightly related with the resolution of problems found with functions, i.e. the fact that a simple convergence condition is not sufficient for a succession of continuous functions to have a continuous function as a limit. This is why the order of appearance of the calculus subject is opposed to the needs that originated its study and formalization. On the other hand, the exploration of the kind of problems that originated the calculus is lost: The drawing and description of the behavior of a curve where it is possible to demonstrate the need of the concepts like monotony, maximum and minimum, concavity, etc.; and the geometrical intuition and calculus important results which emerge from the resolution of curves' problems are neglected again. Resuming, it can be stated that the content and historical order of the studies in the calculus course developed between the XVI and XIX centuries is as follows: Polynomial variations and a polynomial derivative; Relation between a tangent to a curve and a derivative (Barrow); Derivative of the algebraic functions and calculus' birth (l'Hopital); Trigonometric and transcendent functions (logarithm and exponential) and their derivative (Newton, Leibniz), development of Taylor and series; General consideration of real functions by Euler, without giving a formal definition of the object function; Study of the problems of convergence of series and continuity (Fourier); Formal definition of real numbers and of derivate in a point as limit (Weierstrass).

From comparing both progressions we find a strong incoherence, which can clear up some of the observed difficulties in calculus teaching. We find two particularly difficult concepts, limit and derivative, determining a lack of comprehension from an important portion of the students, above all in those careers that are not only scientific (i. e. engineering and social sciences). Obviously the concept of limit was not absent in the mathematics before Weierstrass, because it was presented since the Greek age in algorithmic and geometrical contexts, or in the study of movement (kinematics) i. e. *anthypheresis* related to developments in continuous fractions, Zenon's paradox, tangent as limit position in a chord. Descartes' analytic geometry has established a bridge between geometry and calculus without reducing both dominions to a unique kind of mathematic thought. A tool which had an important historical role, the reasoning of geometric form aside, was tabulation: to calculate results, numerical value tables were used. Moreover, until the computing age, the use of trigonometric and logarithmic tables, were taught.

Some guides, for the differential calculus teaching, result from the study and analysis of history. One guide is that, must not be constructed the first differential calculus course over a previous knowledge of formal definitions of the function concept and of the notion of limit, which emerged at the end of their creation. But to be supported on the numerical calculus stages (e. g. interpolation) and on the algebraic calculus (e. g. Newton's binomial and development of $a^n - b^n$) and even to exploit ideas coming from geometrics or the movement study.

"Even if education has other means than to reproduce the meanders of history, to give sense in concepts and theories which it teaches -we could not afford it- it is clear that we must know how to draw the lessons of these epistemological characteristics and accept that calculation can spread and help to construct a mathematical world, even though its objects are not, and can not be, perfectly defined yet" (Kahane, 2002).

3. **The experimental project of teaching calculus in Mexico**

In order to develop the experiment, a permutation on the subjects of the traditional study program and use of technology has been proposed. The change in mathematics education, with the introduction of the technology, is today something unquestionable and makes a considerable challenge for the educators. But not only introducing a new form of teaching, as efficient as for instance the ACE cycle in cooperative learning in the C4L project, 2004, can not produce the desired improvement, without a correlative change in the cognitive way of approach.

From this point of view, two realizations seem to us essential: To understand that computation in calculus is different from the previous algebra, by the game which it institutes between "local " and "global", to understand that it inserts in a fundamental manner the notion of order of magnitude (Kahane, 2002, p. 241, translated from French by the authors).

This is why visualizing a differential calculus course without the use of technology would make the teacher unable to take advantage of one of the most important resources he can count on nowadays. But also it would be naive to think that the technology use would by itself resolve all the teaching and learning problems. It is advised in this sense that the teacher will have to be careful on the hidden mathematical processes and the new representations of formulas, number and data produced by the software (Lagrange 2005). Our position is then for using technology under a careful didactic scheme. So we have introduced essentially two kinds of programs in a differential calculus course, the first has didactical scenarios which simulate a natural phenomenon. These scenarios, where the student is able to manipulate things, are created in Cabri and Java in order to be later converted in applets of free use on the net. The second kind, it’s a tutorial systems which share the teaching job with the teacher.

For the first part of the course which corresponds to basic concepts of function, dependent and independent variable, parameter and equation, the project of concrete action, named “Project pulleys”, was designed. This project has three applets simulating the movement transmitted by a pulley. Additionally we create a whole work environment for the students, including: Directed instructions for the teachers with the description of the objects of study, the necessary time prevision and organizational instructions; working instructions for the students, Questionnaires, with presupposed spaces, for the students who are conducting the activity; Interactive files (mathematical laboratory), with the inclusion of programs in Applet form which run through a Java motor in any navigator.

The sheets of question-answer are useful for teachers, in order to control the student’s activities, as well as for the students to be able to see, through the interactive files, the directions where to lead their experiments. The activity addresses the students to consider the fundamental ideas for the study of functions, which are independent variable, dependent variable and parameter, as well as the concepts of domain and range of a function. Globe project was created with these same characteristics (Cuevas, Moreno & Pluvinage, 2005). This scenario provides the student with virtual scenario, where we have a balloon tied to the bottom of a cylinder with certain amount of liquid. The student must choose a certain amount of liquid, at the beginning, and starts to inflate the balloon. The problem is to find the balloon’s radio when it is tangent to the liquid’s surface. The student and the teacher propose the equation through a guided activity of a questionnaire, resulting in a cubic polynomial and in order to solve it, the system CalcVisual is used.

CalcVisual (Cuevas and Mejía, 2003; Martínez, 2006) proposes a new kind of learning environment in mathematics, it is called New Intelligent Tutorial System (NITS) and it inherits part of the traditional architecture of an Intelligent Tutorial System (ITS), but it weakens the two fundamental paradigms which according to our approach have impeded the development of the ITS. The first modification of CalcVisual consists in that the tutorial module, does not seek the teacher's total substitution. We intended to design a system that shares with the teacher the responsibility of a course. The second contribution corresponds to modify student's model module. In this case we propose a model of the student's statistical error (SES). It does not mean to have a virtual student who is susceptible to do all the possible student's errors and successes, on the contrary, it is the creation of a structured bank of data that contains the most frequent errors made by students when solving a problem. In order to live the experience with CalcVisual (see figure 1), two hours were devoted to work attending the lesson and using the traditional resources (blackboard, chalk, diagrams, exercises in pencil and paper, etc.) with the teacher. The other two, were used in working with CalcVisual without the teacher. In some cases they acted as an assistant teacher but like observer in the Keskessa’s sense. This is, observer's function should be to speak with the teacher essentially about the organization of the activity of the student and its progress related to a setting previously established (Keskessa, 1994). Laboratory work was designed as collaborative; two students per computer attended the Vigotsky’s socio-cultural theory that states the importance of the environment for knowledge acquisition. He mentions the teacher works as a mediator between knowledge and student, in such a way that when incorporating the collaborative work among students, each one of them, in different moments, takes that mediator role. Thus, this work scheme, where two students interact in a computer, allows interchanging questions and reducing the zone of proximal development. The latter allows covering the content of curricula faster, easier and in the dynamic of an efficient work..

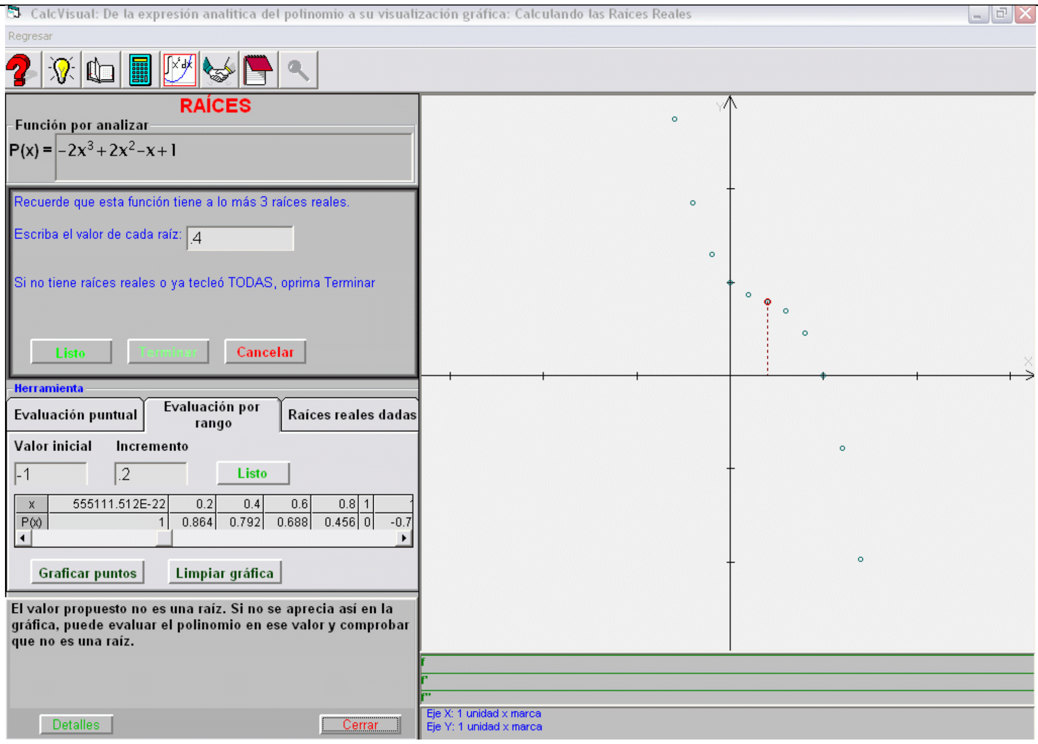

Figure 1. CalcVisual Interface, note that the main menu has been modified.

While the design of a local experiment authorizes some adaptations of the didactic proposal to the public and to the context as they appeared during the application period, the design of a curricular proposal that has robustness sufficient to experience it at a more important scale, supposes a strict observation of didactic principles. Hence, we have selected the following:

1. Due to the students' heterogeneity in previous mathematical knowledge, application of a spiral teaching accordingly with the principle of proximal development (Vigotzky) in order to avoid remedial courses.

2. Designing and implementing modeling activities to promote the concepts, in accordance to the RMI scheme (real mathematical instruction), in the form of concrete action projects (Cuevas &Pluvinage, 2003).

3. To favor the control of the students over their own learning, systematization of the use of diverse registers of semiotic representation (tabular, graphic, algebraic), with which the functions are represented and promote the conversions between registers in both ways.

4. First observed results

For the first time in 2003, the CalcVisual software is installed in a calculus course, in a high level career. The aim to conjugate using this NITS with applying exercises in classroom produced improvement in the calculus, its study resulting more conceptual than operative, although the mathematical concept is not formalized before a further stage.

Before the use of the NITS in the institution the failure rates were of 80%. The traditional pattern of evaluation consists in: exam written 50%, homework 30%, activities works 10% and participation 10%, so that all students that approved the course should have punctuation in each item. Gradually through the last three years this failure index has diminished to 25%, maintaining the traditional pattern of evaluation. Given that the evaluation depends on the teacher's approach even when the parameters settle down, we decided to carry out a quantitative analysis in statistical terms, to confront the results obtained in a diagnostic exam (pretest) with the results of a posttest and find out if they show an improvement. Both exams were applied to LIA (administrative engineering) and ICO (computing engineering) groups; and on time, they will show that in some aspects a promotion is observed in the learning of the students. Although many of the questions have to do more with pre-calculus than with the characteristically contents of the calculus, it is observed that when developing activities for the promotion of the basic concepts of calculus, the algebraic handling improves later on, without need of special courses. If we examine each case, we see that the use of the tools provided by the NITS facilitates the activity of algebraic factoring by means of the tools that the software presents, which is its purpose. A known problem for the college students is the deficiency in the algebraic handling and in particular in the factorized polynomials. When developing activities in each register of semiotic representation (algebraic writing, table and graph) for seeking roots of a real valued function and interpreting them like in a decomposition of binomials, the system allows the student to overcome the algebraic obstacles.

5. Conclusions

The use of the technology lowers didactic limits shows flattering results when diminishing the time of present class substantially, besides reducing the number considerably of having failure. Additionally, in the last years we have diminished in other courses of mathematics of the same plan of studies this index of having failure. At the present time, this project has extended to six universities more than the country, adding proposals of curricular change that include proposals of court epistemological; we are while waiting for the results that they are obtained

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